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Q measurements at microwave frequencies

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Q MEASUREMENTS AT MICROWAVE FREQUENCIES

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

ABSTRACT

Various methods of measuring microwave cavity Q are discussed, with particular attention being given to their potential accuracy and simplicity.

A heterodyne marker system using a calibrated crystal and a comparison system are described in detail.

INTRODUCTION:

The Sperry Gyroscope Company has a contract with the Army Signal Corps to measure the surface resistivity of a number of materials with a variety of surface finishes in the frequency range of 1 to 40 Kmc. The materials will be standard alloys of such metals as iron, steel, nickel, stainless steel, brass, silver, nichrome, manganin, silicon, etc. The specific alloys to be tested will be selected on the basis of their probable usefulness. The purpose of this program is to obtain sufficient data so that common materials previously neglected can be used intelligently in the design of microwave components.

The experimental work consists of a study of measuring techniques followed by the actual resistivity measurement at ten frequencies in the above-mentioned range.

Methods of Measuring Surface Resistivity:

1. Attenuation Methods:

Surface resistivity can be obtained by the measurement of the attenuation of a length of waveguide fabricated from the material (Reference 1)*.

2. Cavity Q Methods:

Alternatively the material may be used to build a simple microwave cavity, the Q of which is measured. The surface resistivity of the sample can then be calculated using

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* Bibliography may be found at the rear of the text.

the Q data.[2] When long pieces of homogeneous rectangular or circular tubing with various cross-sectional sizes are available in the desired materials, the attenuation method is a simple and straightforward way of measuring surface resistivity. However, the cavity Q technique has the advantage of requiring a much smaller quantity of the sample material, and for this reason it is to be used.

Beck and Dawson used a coaxial line cavity for their surface resistivity measurements at 9 Kmc. (Figure 1)[2]. The heart of this type of cavity is the coin silver tube which becomes the outer conductor of a coaxial line when the specimen is inserted. By varying the specimen lengths, the resonant frequency of the cavity is changed. Thus, many materials may be tested over a large band of frequencies without altering the basic cavity. When the wave propagated down the coaxial line comes to the end of the specimen it sees a circular waveguide operating below cutoff. The tube is made long enough so that essentially no energy can leak out of the ends. The ends are left open to permit insertion and removal of the specimens, which are wires supported at voltage nulls by polyfoam beads. The ratio of tube to specimen diameter is made large so that almost all of the loss occurs in the specimen. The outer conductor loss and the coupling Q can be measured so that the value of the unloaded Q, required for the surface resistivity calculation, is obtainable from the value of the loaded Q. This type of cavity is excellent for this work

except at the very short wavelengths, where the tube dimensions are so small that tolerances, eccentricities, etc., become troublesome. Two of these coaxial type cavities have been constructed. One, having coaxial feed lines, is designed to operate at frequencies of 1 to 4 Kmc. The other, having RG-50/U (1-1/2" x 3/4") waveguide feed lines, is designed to operate at frequencies of 4.5 to 8.5 Kmc. Rectangular cavities for the frequencies between 8.5 to 40 Kmc will be used. The value of Q for the different samples will range from 200 to 2000 at a frequency of 7.8 Kmc, and the power transmission ratio will vary between -20 and -50 db. It is hoped to determine the unloaded Q with an accuracy of ±5 percent.

Methods of Measuring Cavity Q:

The Q of an oscillatory system is defined as follows:

$$Q = \frac{\text{total energy stored in the system}}{\text{energy dissipated per radian}} \tag{1}$$

A cavity has an infinite number of modes of oscillation, and can be represented by an n-mesh Foster equivalent circuit with $n = \infty$ [3]. However, if the modes are well separated, and the cavity losses are not too high, in the region of one of the resonant modes the susceptance contributions of the other modes can be ignored and the equivalent circuit reduced to a simple RLC circuit. This leads to the use of the definition of Q so common at the lower frequencies. (Appendix I).

$$Q = \frac{f_o}{\Delta f}\sqrt{\frac{1}{P} - 1} \tag{2}$$

f_o = resonant frequency

Δf = frequency difference between the two points on either side of f_o where the transmitted power has dropped to P times the value at resonance.

P = a number between 0 and 1 which defines the power level at which the Δf is measured.

1. Pulse Decrement Method:

The pulse decrement method of measuring Q is based on the first definition[4]. When a pulsed r.f. signal at the resonant frequency is passed through a cavity, the build-up and decay of the pulse is a function of the Q. From equation 1 it can be shown that the decay is in accordance with the following relationship:

$$W = W_o\, e^{\frac{2\pi f_o t}{Q}} \tag{3}$$

W = energy in the system at time = t

W_o = energy in the system at time = o

For the favorable condition of a Q of 1000 at a frequency of 1 Kmc the rate of decay is 27.3 db/microsecond. This presents a difficult measurement problem.

All of the other known methods of measuring Q depend upon making observations at a frequency differing from the resonant frequency by an accurately known amount. As can be seen from equation 2, the accuracy of the Q determination can be no better than the accuracy of the measured frequency difference. If the frequency difference is to be obtained by subtraction of two measured r.f. frequencies, the measurement accuracy must be of the order of 2Q times the required accuracy of the Q determination. The observations made at these points may be either relative power transmission, impedance measurements, or the shape of the response curve. As input impedance at microwave frequencies is measured with an impedance meter (standing wave indicator), this also reduces to a relative signal level determination.

Relative signal levels of low power signals at microwave frequencies may be measured by a number of means, the most common which make use of crystals, bolometers or calibrated attenuators. The calibrated attenuator of course depends upon the use of a more fundamental method in its calibration. A microwave attenuator of small total insertion loss may be calibrated against a precision IF attenuator to an accuracy several times as great as the accuracy of the standard itself by using the shorted line input standing wave ratio method[5]. If the attenuator is of the metallized glass type and of a good mechanical design, it should be

possible to read attenuation differences to a few hundredths of a db. An error of 0.04 db in the level measurement will produce an error of about 1 percent in the Q determination. It has been claimed that barretters can be made to exhibit a change in resistance which is directly proportional to the change in power level (square law operation) to within ±1 percent, although this is open to some question. They have an upper frequency response of only about 500 cps however, and a sensitivity of about 40 Mv per milliwatt[6]. As they are operated with a heavy d-c bias, the match to the line does not change appreciably with changing power level.

Crystals are much more sensitive than barretters. For instance, a 1N32 crystal with a 3.9 K ohm load delivers the same voltage output with about 13 db less input power. They are not, however, as reliably square law devices as are barretters, and they become mismatched with relatively small changes in power level. It has been found that the type 1N32 is one of the better crystals as far as consistency of detector law is concerned. Figure 2 shows the results of some crystal studies at Sperry. It was the belief of the investigator that the deviation from square law at the low power actually represented an error in his measurement technique.

2. Impedance Methods:

The types of Q determination which make use of impedance measurements require that the cavity being measured terminate the line. They are therefore not applicable to this problem.

3. Inflection Point Method:

A method of Q measurement known as the inflection point method is being developed at Sperry. It makes use of the fact that the second derivative of the response curve vanishes at a point on either side of the resonant frequency. As has been pointed out by Roder and others, a tuned circuit will act as a demodulator for frequency modulation if the center frequency of the signal is somewhat displaced from the resonant frequency of the tuned circuit[7]. Due to the curvature of the response, in general, the demodulation is accomplished by harmonic distortion of the modulating frequency. The amount of second harmonic distortion should be at a minimum when the carrier frequency is centered at the vanishing point of the second derivative. A highly selective amplifier tuned to the second harmonic of the modulating frequency is used as the indicator following the detector. Two modulating frequencies may be used and the selective amplifier tuned to the sum or difference intermodulation product. The large amount of noise free gain required to obtain a precise measurement, the required stability and purity of the modulated and modulating signals, and the dependence upon the law of the detector are limitations of the system as it now exists. If a linear detector is used, the inflection points are $f_o/\sqrt{2Q}$ cycles apart but, if detection law is two the inflection points are $f_o/\sqrt{3Q}$ cycles apart. This

means that a crystal detector can introduce an error in this determination just as when it is used as a power level indicator. The use of a barretter as a detector is squeezed between a low upper cutoff frequency and the difficulty of obtaining the required selectivity at low frequencies, with the added disadvantage of being much more insensitive than a crystal. Due to the smaller "Δf", the accuracy of the absolute frequency measurement before subtraction must be even greater than with more conventional methods.

4. Sweep Frequency Marker Methods:

Several methods have been developed for measuring the "Δf" directly, rather than as a difference between two quantities[8]. All of these methods make use of a swept frequency oscillator whose frequency excursion includes the response of the cavity under test. If the output of a detector following the cavity is applied to the Y axis of an oscilloscope and a voltage proportional to the frequency sweep is applied to the X axis, the resulting pattern will be the response of the cavity as distorted by the detector, the Y amplifier, and non-correspondence between the frequency sweep and the voltage sweep to the cathode ray tube. Frequency markers may now be applied to the response pattern and adjusted to occur at identifiable points. These markers may be generated by heterodyne or modulation methods.

In the modulation method a sample of the swept oscillator frequency is mixed with another oscillator signal of about the same frequency, which is frequency modulated at a rate equal to the "Δf" or a sub-multiple thereof. The output of the mixer is amplified by a zero beat audio amplifier and the output applied to the X, Y or Z axis of the oscilloscope. The "Δf" determination then reduces to the determination of the frequency of the modulating signal.

In the heterodyne method the sample is mixed with a stable CW signal and the output amplified by a highly selective communications receiver. The receiver has an output, when tuned to a given frequency, whenever the swept oscillator is that frequency above or below the stable oscillator. The frequency difference between the markers is then twice the frequency to which the receiver is tuned.

5. Sweep Frequency Comparison Methods:

The response curve as displayed on the oscilloscope may be compared with another response curve displayed on the same tube. The second response curve may be generated in either a single detector or a two detector system (Figures 3 and 4).

In both systems the frequency excursion of the swept oscillator is moved intact to the IF frequency. As the mixer is a linear device the relative power during the frequency sweep remains the same on the IF side of the mixer as it is on r.f. side. To obtain coincident response curves, the Q

of the IF circuit must be adjusted so that its Q is equal to the cavity Q times the ratio of IF to r.f. resonant frequencies. The Q of the IF standard may then be measured by conventional low frequency methods. Different distortion in the two channels is a possible source of error in the two detector method.

The single detector method attempts to overcome the above difficulty by re-combining the signals ahead of the detector or any nonlinear device[9].

Selection of the Methods to be Used:

Measuring frequencies to an accuracy of one part in 100,000 implies a crystal controlled standard. As no such frequency standard was available to this group in the foreseeable future a sweep frequency system had to be used. It was decided to try a marker system first then check the results with the single detector comparison system. Heterodyne markers were chosen, because it was felt that it would be easier to stabilize the beating oscillator if it were not modulated.

Heterodyne Marker System:

The equipment shown in figure 5 was assembled. The sweep generator is from a microwave spectrometer system modified to give a sweep with a maximum amplitude of 120 volts and a minimum recurrence frequency of 8-1/3 cps. The klystron power supply is a Microline Model 444 with the impedance of

the modulation input circuit increased to 1 megohm to decrease the loading on the sweep generator. The blocking oscillator tube of the internal modulation generator in this unit was also removed to decrease noise. The swept oscillator is a SRC-43X klystron having a power output of about 1 watt. The wavemeter is a Microline model 208 with an absolute accuracy of 0.1 percent. The variable attenuator is a Microline model 216 with an attenuation range of 2-20 db. The directional coupler is a Microline model 209 which was calibrated at 7.76 Kmc using a wattmeter bridge. The coupling was found to be 24.3 db. The detecting section is a Microline model 206 which has an adjustable short behind the detector mount. Either a crystal or a barretter may be used in this section. The preamplifier is the first stage of a General Radio model 714A amplifier with the input coupling capacitor increased from 0.04 microfarads to 2 microfarads. This reduces the lower cutoff frequency to about 0.04 cps. The voltage gain is about 20. The cathode ray oscilloscope is a Dumont type 304H having a 5CP1-A tube (medium persistence screen). The maximum vertical deflection sensitivity is 3.5 millivolts per inch. The input amplifier tubes were picked to obtain the minimum hum pickup with maximum gain when the amplifier is properly balanced. The coaxial mixer is a Microline model 338. This is not the correct mixer for the 7 Kmc band but it is satisfactory as

the signal is well above the noise level. The beating oscillator is a 2K44 operated at the very top of its frequency range. Although a well regulated power supply is used for the 2K44, it was found necessary to add a two section RC filter to the repeller circuit. Each section consists of a 1 megohm resistor and a 0.5 microfarad capacitor. The communications receiver is a National HRO-7R.

1. Level Determination:

It was originally planned to obtain and calibrate a precision attenuator, however, no such attenuator was available in RG-50/U waveguide and none would be available for several months. It was then decided to use a 1N32 crystal and operate in the square law region.

The reading of the oscilloscope is the source of measurement error other than system error in the level determination. An effort was made to evaluate this error by putting a 60 cycle signal on the oscilloscope and adjusting the signal level for a 3" deflection. The signal was then measured with a Ballantine model 300 VTVM with a stated accuracy of ±2 percent. From six observations, the mean deviation from the mean value was 0.75 percent. The greatest deviation was 1.25 percent. It was concluded that level readings could be made on the oscilloscope with a precision of at least ±2 percent.

The sources of system error in the level determination are the following:

1. Crystal not exhibiting a linear power input voltage output characteristic.
2. Power variations over the frequency sweep.
3. Nonlinearities in the amplifier.

Toward the end of the work period after the failure of the independent method, it was decided to calibrate the crystal under conditions as nearly like its actual operating conditions as possible. To do this the SRC-43X was operated without modulation into an impedance meter followed by a soldered short circuit. About 3 or 4 db padding was used between the tube and the impedance meter. The probe output was applied to the waveguide detecting section through a double stub tuner and a waveguide to coaxial adapter. The crystal load consisted of a microammeter and enough additional resistance to bring the total load up to 3.9 K. The efficiency of the short was checked by adjusting the probe coupling for 50 microamperes at a maximum point, then exploring the nulls with a meter capable of detecting a current of 0.005 microamperes. No deflection could be noted at a null. Assuming the crystal to be approximately a square law device, the power standing wave ratio in the guide is greater than 40 db. The short can thus be considered to be perfect. The relative power picked up by the probe when the line is terminated in a perfect short is:

$$P_R = \frac{1}{(1+G_p)^2 + (B_p+\tan\theta)^2} \tag{4}$$

P_R = relative power picked up by the probe as a function of its position along the line

G_p = conductance introduced into the line by the probe (coupling coefficient of the probe)

B_p = susceptance introduced into the line by the probe

θ = distance between the probe and a point on the line where the perfect short has been transformed into a perfect open circuit

If

$$B_p = 0 \quad \text{and} \quad G_p \ll 1$$

$$P_p = \frac{1}{1+\tan^2\theta} = 1/2 + 1/2 \cos 2\theta \tag{5}$$

B_p makes the curve asymmetrical and G_p introduces a vertical distortion[10]. G_p was measured as the ratio of the incident power to the power picked up by the probe with a wattmeter bridge after adjusting the double stub tuner for a maximum transfer of power. G_p was found to be -40 db for the largest crystal current to be measured and therefore does not affect the shape of the curve by a measureable amount. With a G_p so very small and the probe

tuned for maximum output B_p should also be small. This was verified by the fact that no distortion of the curve could be detected. Current readings were taken over two guide wavelengths starting with peak readings of 25, 10 2.4 and 0.5 microamperes. The short behind the crystal was adjusted for a maximum reading at the peak values at the start of each run and was not touched after that. Figures 6, 7, 8 and 9 are the curves of current (voltage) vs. power level. The microammeter sensitivity was adjusted to make each peak value a full scale reading. The meter scale was 100 millimeters long graduated with millimeter marks so it could be read to a precision of ±0.2 percent of the full scale reading. The impedance meter could be reset with a precision of ±0.05 mm which represents ±0.34 degrees at the guide wavelength. The guide wavelength could be obtained to a high precision by measuring the distance between corresponding points on the steep part of the curve several half wavelengths apart but it was difficult to locate the nulls as they are quite broad. However, if an error is made in locating the null, all of the data from points on the generator side of a null should fall on one side of the true curve and all of the data from points on the short side of the null should fall an equal distance on the other side of the true curve. The observed data can thus be corrected to eliminate this error. It would

appear that an error of as much as 20 percent will be made at a high signal level and 5 percent at a relatively low signal level if it is assumed that the crystal will give a linear power to voltage conversion. Using these correction curves the error from this source should be reduced to within ±1 percent.

A source of error is the fact that a klystron does not supply constant power as the frequency is shifted. A correction for this effect is obtained by centering the response on the mode before starting the determination then measuring the relative drop of the mode in the interval of the chosen "Δf". This can be done with a barretter or a calibrated crystal to the same precision that the response curve is read.

The gain linearity of the preamplifier oscilloscope combination was checked using a Ballantine model 300 VTVM and a 60 cycle signal. For most gain settings no distortion could be observed however, when distortion is observed at a particular gain setting it can be included as a correction to the computation so that errors from this source should be within ±2 percent.

2. "Δf" Determination:

To measure the precision of the "Δf" determination, a "Δf" of about 6.8 Mcs. corresponding to a certain interval

on the oscilloscope was measured 10 times by varying the receiver tuning. The mean deviation from the mean of these readings was 0.2 percent. The maximum deviation was 0.6 percent.

The sources of system error in the "Δf" determination are the following:

1. Different delays in the detector and marker channels.
2. Frequency calibration errors in the communication receiver.

The heterodyne marker system depends upon the identification of a moment of time (a position on the cathode ray scope) with a frequency and a transmission ratio. If the time delays in the marker channel are different from the time delays in the response channel an error will exist if the frequency sweep is not linear with respect to the time sweep. It was suspected that the low sweep speeds used would make any time differences unimportant. This was proven by coupling a small amount of the beating oscillator energy into the transmission line ahead of the cavity so that a small disturbance was observable on the response curve. When this disturbance was centered at the peak of the response it indicated that the beating oscillator was operating at the resonant frequency of the cavity. Any time delay difference would then show up as a shifting of the center of the marker pulses with respect to the center of the response curve. No shift was observed.

The communcations receiver tuning dial was calibrated using a recently checked General Radio model 605B signal generator. Stated frequency accuracy of this unit was ±1 percent. This relatively crude method of calibration was used because a crystal controlled heterodyne frequency standard was not available for general use.

Combining the level and "Δf" measurement precisions, the expected precision of the loaded Q determination independent of system errors should be within ±2.0 percent. To obtain an experimental check of the over-all precision, a sample with an unloaded Q of about 2590 at a frequency of 7815 Mcs. was measured nine times. The mean deviation from the mean of the nine runs was 1.1 percent and the maximum deviation was 2.2 percent. Another sample with an unloaded Q of about 800 gave a mean deviation from the mean of 21 readings of 1.4 percent and a maximum deviation of 3.9 percent.

3. Data Conversion:

Even if the loaded Q of a cavity is known to a very high accuracy, the following are sources of error in converting the data to surface resistivity values.

1. Errors in measuring the transmission ratio.
2. Center conductor not positioned axially.
3. Center conductor not centered radially.
4. Losses in the polyfoam beads.

The power transmission ratio is obtained by measuring the power at the directional coupler then replacing the

detecting section with a barretter mount and measuring the power at this point. The coupler has been calibrated using the substitution method to an estimated accuracy of ±0.4 db. When the transmission ratio is about the same as the coupler ratio (24.3 db) the estimated accuracy of this determination is about ±0.5 db. The unloaded Q of this cavity is obtained from the loaded Q by the use of the following relationships(2).

$$Q_o = \frac{Q_L}{1-2\sqrt{T}} \qquad (6)$$

Q_o = Unloaded Q

Q_L = Loaded Q

T = Numerical power transmission ratio

When the power transmission ratio is -20 db the above transmission ratio error introduces an error of about ±1.4 percent in the unloaded Q computations. The error is about ±0.8 percent for -30 db and about ±0.1 percent for -40 db.

Beck and Dawson state that small axial misalignments of the center conductor will result in different values of loaded Q but, as the power transmission ratio is also measured for the same wire position, the unloaded Q values will be the same. To experimentally verify this fact, measurements were made with the sample displaced from the position of maximum coupling. When the transmission ratio was reduced from the maximum by 0.7 db, the loaded Q values differed by 2.0 percent but the unloaded Q values differed by only 0.7 percent. As the

transmission ratio can be maximized to within much closer limits than 0.7 db it can be concluded that there is no measureable error from this source.

It was planned to check the effect of radial eccentricities on the measured values of loaded and unloaded Q but time did not permit. However, it is believed that, as the ratio of outer conductor to inter conductor is quite large, small eccentricities will have no appreciable effect on the Q values or on the conversion of the Q data to surface resistivity values.

As polyfoam is only about 5 percent polystyrene and the beads are located at the voltage nulls the losses in the beads should be quite low. As a rough check, the beads were placed at the voltage maximum and the Q of a high Q wire was lowered only 3.6 percent at a frequency of 7.8 Kmc. This would seem to indicate that the effect of the beads at the voltage nulls is much less than the measurement precision of the system.

It is expected that the total system error, considering all of the above sources, should fall within ±3 percent and the absolute error of the average of a number of determinations should fall within ±4 percent of the true value. The measured surface resistivity of a very smooth sample of a pure metal will be compared with the theoretical value as a final check of the over-all system accuracy.

Single Detector Comparison System:

In order to obtain an independent check of the value of loaded Q an attempt was made to set up the single detector comparison system. The response curve of a circuit with a Q of less than 10 starts to differ in shape from a response curve of a high Q circuit. As many of the materials give a loaded Q of 700 or less at 7 Kmc the IF standard circuit must operate at 0.1 Kmc or above. Even though distortions in the common amplifier affect both signals in the same way it is desirable that the bandwidth of this channel be several times greater than the half power "Δf" of the IF standard circuit. To bypass the formidable gain beamwidth problem in the IF amplifier, the output of the switch was fed directly to the detector and all gain was obtained at audio frequencies.

For the IF switch a commutator for the signals and a shorting switch to replace the gas tube in the sweep generator were ganged to a motor shaft. Contact noise in the signal commutator, however, proved prohibitive so an electronic switch (figure 10) was designed. A common plate circuit was used in an effort to equalize the distortion in the two channels. The response curves of this switch (figure 11) indicate that its performance is satisfactory. Figure 12 shows the IF standard circuit. Loose coupling was used between the mixer and the standard so that a signal generator could be substituted for the mixer crystal for calibration without affecting the

loaded Q. The voltmeter used for calibration was to remain attached at all times. The resonant frequency shifted 0.5 Mcs. during a change in Q from 10 to 100. The voltage transfer ratio at resonance from mixer input to switch input was 0.04. When work on the system was halted due to lack of time, difficulties were being encountered in obtaining sufficient signal from the mixer in the comparison channel, and in removing 60 cycle amplitude modulation from the beating oscillator.

With a more stable and powerful beating oscillator more attention to the mixer design and with more hum free audio amplification it is believed thatthe system could be made to work even at this unfavorable frequency to Q ratio.

CONCLUSIONS:

A survey of the various methods of measuring cavity Q was made and it was concluded that the heterodyne marker system with a calibrated attenuator to measure relative power was the simplest method having a potentially high accuracy. A system using a calibrated crystal was assembled and Q measurements having a repeatability of ±1½ percent were made.

An independent check of the accuracy of the system has not been made.

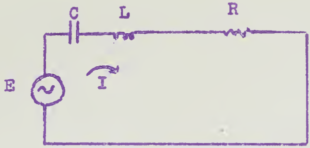
Recommendations:

It is recommended that a stable variable attenuator of low total insertion loss be obtained, calibrated and used for a check of the level determination accuracy.

APPENDIX I:

Deviation of the equation for the response curve of a tuned circuit.

Consider the simple RLC equivalent of a high Q loaded cavity.



The output load resistance is included in R so that the power response is proportional to I^2

$$I_\omega = \frac{E}{Z} = \frac{E}{R+j(\omega L-\frac{1}{\omega c})} \tag{1}$$

Where ω is any angular frequency. The energy stored in the circuit is:

$$W_s = L\ I^2 \tag{2}$$

The energy dissipated per radian is almost exactly:

$$W_D = \frac{R\ I^2}{2\pi f_o} = \frac{R\ I^2}{\omega_o} \tag{3}$$

From Equations (2) and (3):

$$Q = \frac{W_s}{W_o} = \frac{\omega_o L}{R} \qquad (4)$$

At resonance

$$\omega_o L = \frac{1}{\omega_o C} \qquad (5)$$

From Equations (4) and (5)

$$Q = \frac{1}{\omega_o RC} \qquad (6)$$

From Equations (1) and (6)

$$I_\omega \alpha \frac{1}{1+jQ\left[\dfrac{\dfrac{\omega^2}{\omega_o^2} - 1}{\dfrac{\omega}{\omega_o}}\right]} = \frac{1}{1+jQ\left(\dfrac{f^2-f_o^2}{f\ f_o}\right)} \qquad (7)$$

As relative power is proportional to the magnitude of I^2

$$P_R = \frac{1}{1+\dfrac{Q^2}{f_o^2}\ \dfrac{(f+f_o)^2(f-f_o)^2}{f^2}} \qquad (8)$$

$$f + f_o \doteq 2f \tag{9}$$

$$(f - f_o)^2 = \delta^2 \tag{10}$$

If Δf is defined as the frequency difference between the two points on either side of f_o having the same power level:

$$2\,|\delta| = \Delta f \tag{11}$$

then from Equations (8), (9), (10) and (11)

$$P_R = \cfrac{1}{1 + \cfrac{Q^2 \Delta f^2}{f_o^2}} \tag{12}$$

and

$$Q = \frac{f_o}{f} \sqrt{\frac{1}{P} - 1} \tag{13}$$

When $P = 1/2$

$Q = \dfrac{f_o}{\Delta f}$ which is the commonly encountered relationship.

APPENDIX II:

General:

All equipment should be operated one half hour before any measurements are taken so that all items will have reached a stable operating condition. Both klystrons should be mechanically isolated from any source of vibration such as blowers, etc. Blowers should not be too close to the tubes as turbulences in the air flow around the bellows may cause frequency modulation of the output. The amplifier in the oscilloscope will have to be balanced occasionally as outlined in the instruction book and the first 12AU7 in the vertical amplifier may have to be selected for minimum hum pickup. The oscilloscope should be calibrated and its linearity checked at three points in each of the two lower attenuator ranges after each major adjustment. The ratio of input voltage for 2 inches signal height to input voltage for 4 inches signal height is designated as "s". Amplitude modulation of the signal from the SRC-43X has been noted at some levels of operating voltage and it should be varied in any effort to eliminate hum. Hum can sometimes be reduced by reversing power plugs in their sockets. The reflector voltage of the beating oscillator has to be varied slowly when trying to locate a mode and all fine tuning should be done with the cavity because of the long time constants in the repeller supply filter.

Great care should be exercised at all times to prevent any damage to the calibrated crystal from either excessive r.f. power or static discharges.

When looking for the resonant frequency of a new sample it must be remembered that there are three variables: oscillator cavity tuning, sample position and detector tuning.

Sample Preparation:

The polyfoam beads should be no more than 4 or 5 millimeters thick and should be secured to the wire with a minimum of low loss coil dope. Two beads can be used 1/4 wavelength from each end of a straight stiff wire. Three or more will be needed for a warped or very flexible wire. The beads should be rolled on edge under a slight amount of pressure to reduce their diameter enough so that they may be pushed into the tube without shifting their position on the wire. A polyfoam pusher should be used when trying to locate the wire at a point of maximum coupling. The pusher should not be left in the tube during actual measurements. At least three samples of each material should be used so that any inhomogeneities may be detected. The sample length may vary between 2 and 5 or 6 half wavelengths. One half wavelength should not be used until it has been verified that exciting the very end of the sample does not give erroneous results. Samples longer than 6 half wavelengths are not necessary and may give trouble due to excessive attenuation of the wave between the coupling hole and the end of the sample. The sample ends should be square and the various lengths of a sample should all resonate within one or two percent of each other but greater precision is not

necessary as the results can be normalized to a single frequency.

Taking Data:

1. Insert the sample to a position of maximum power transmission. Do not use the end position.
2. Peak up the detector tuning.
3. Position the wavemeter notch at the very peak of the response curve. Record the resonant frequency and do not change the wavemeter setting.
4. Substitute the mode pattern for the response and adjust the SRC-43X cavity tuning until the wavemeter notch is at the center of the mode pattern. This centers the response on the mode. If the crystal is used for the detector the barreter may be used for the mode monitor or vice versa. When the barretter is used in either position the sweep frequency should be at a minimum and the sweep amplitude just enough to return the pattern to the base line on both sides. This precaution is necessitated by the low upper response frequency of the barretter.
5. Replace the response on the scope and adjust the pattern to 4" peak to base with the wavemeter detuned. If the crystal is being used as the detector keep the peak voltage less than 0.04 volts or set it to exactly 0.098 volts so that the crystal calibration curves may be used. The peak level can be obtained from the oscilloscope calibration data.

6. Adjust the horizontal gain to make the pattern about 3" wide at the half amplitude point.

7. Tune the wavemeter to the peak of the response curve, then replace the mode pattern. Do not touch the horizontal gain but using the horizontal position control locate the base line and adjust the mode amplitude to some convenient value observing the same precautions as in 5. Using the wavemeter notch as a reference, again center the mode and note the amount of drop-off in the 3 inches selected on the response curve. The ratio of the amplitude at the selected point to the peak amplitude is designated as "m" . The crystal correction factor is not applied as the drop-off should be a relatively small amount.

8. Return the response to the scope. De-tune the wavemeter. Check the 4" and 3" measurements setting the extremeties of the 3" width at heavy lines on the scope.

9. Replace the response with the markers from the receiver being careful not to touch horizontal gains or position controls. The vertical controls may be used. Tune the beating oscillator and receiver until the markers appear on the scope. Adjust r.f. gain, audio gain, selectivity and phasing for the cleanest markers and steepest leading edges. The leading

edges are on the right as the sweep is from right to loft. The leading edges should be placed at the heavy lines selected in 8 using the oscillator tuning control and receiver tuning. Read the receiver dial and record for Δf twice the frequency from the receiver calibration curves.

10. The value of "P" in equation (2) is the product of "s", "m" and "c" where "s" and "m" are defined above and "c" is the power level from the crystal calibration curve corresponding to voltage level "s".

11. With the response curve on the screen, decrease the sweep voltage amplitude as far as possible. With minimum vertical oscilloscope gain and enough horizontal gain so that the hum gives a 2" sweep, adjust the repeller voltage of the SRC-43X until the line is horizontal. The SRC-43X is then supplying a signal with a center frequency equal to the cavity resonant frequency and with a very slight amount of frequency modulation. With a wattmeter, measure the power at the coupler and at the position of the detecting section. The ratio of these two powers times the coupler ratio is the transmission ratio at resonance.

12. Unloaded Q is obtained by use of equation (6).

13. Q can be normalized in frequency by making use of the fact that Q is proportional to the square root of frequency.

14. The surface resistivity can be approximately normalized in temperature by making use of the d-c coefficient of thermal resistivity.

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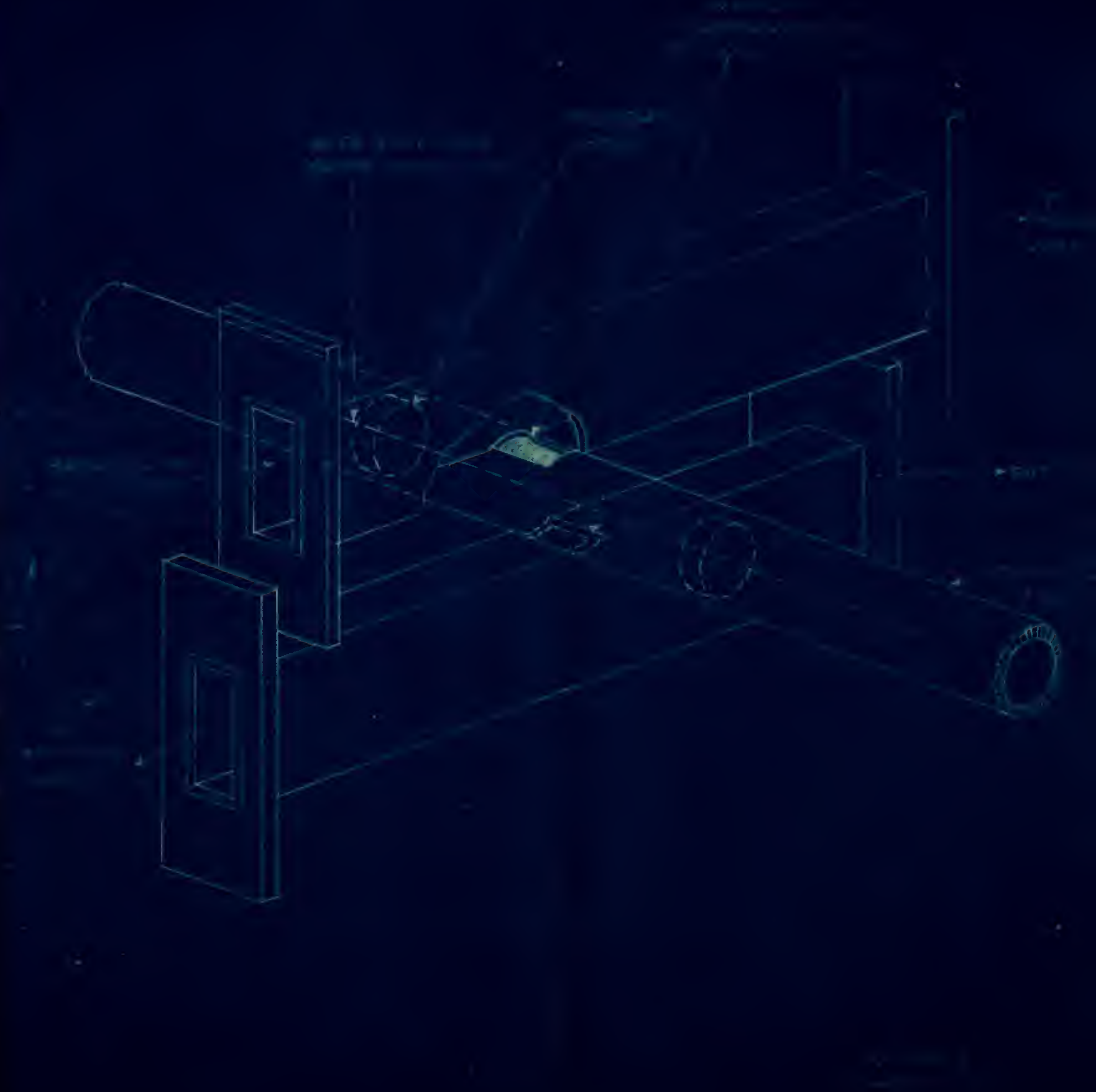
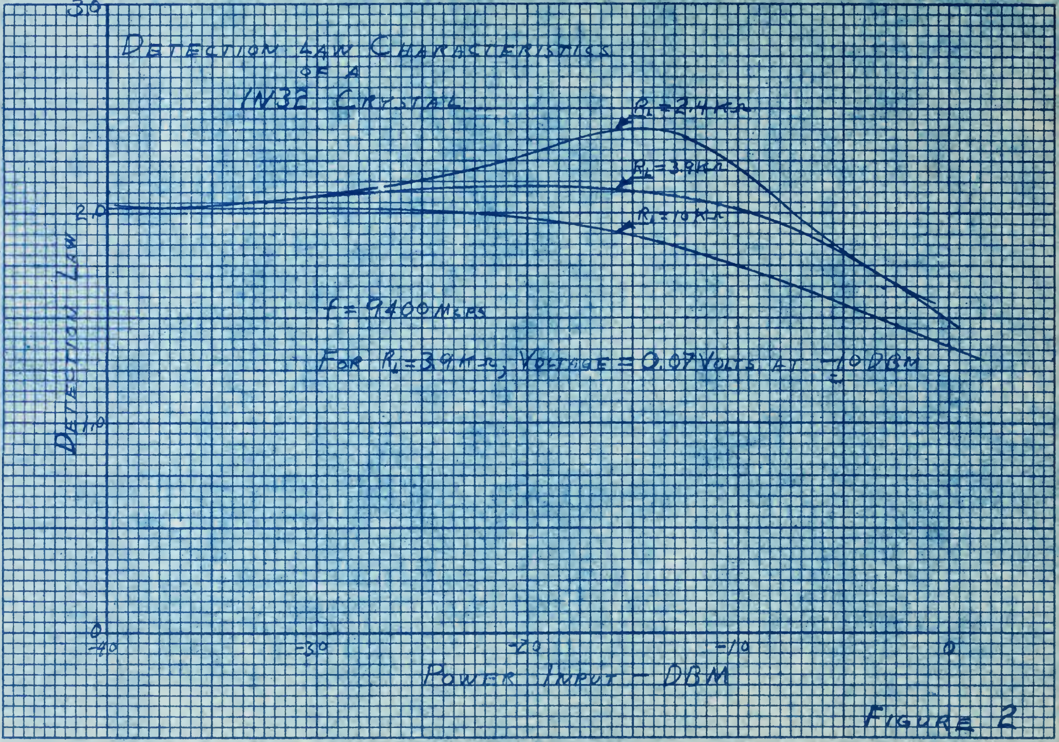
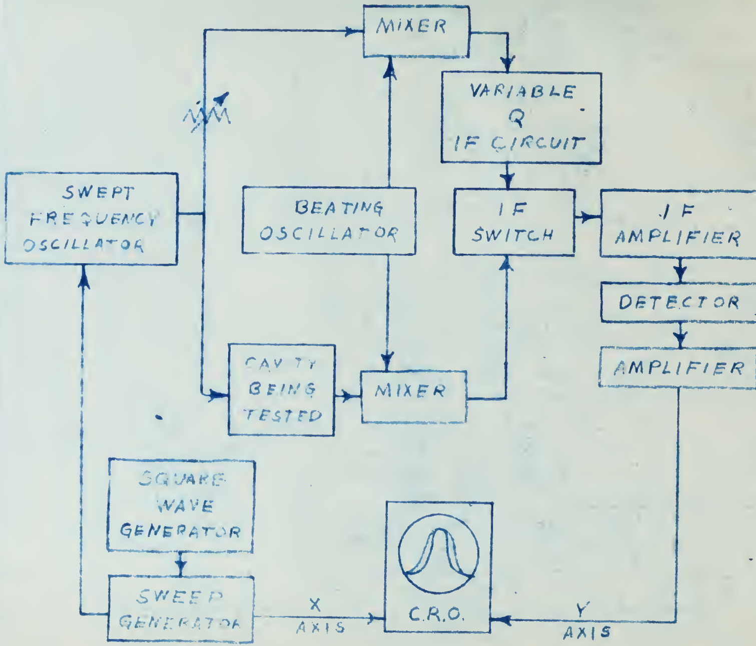




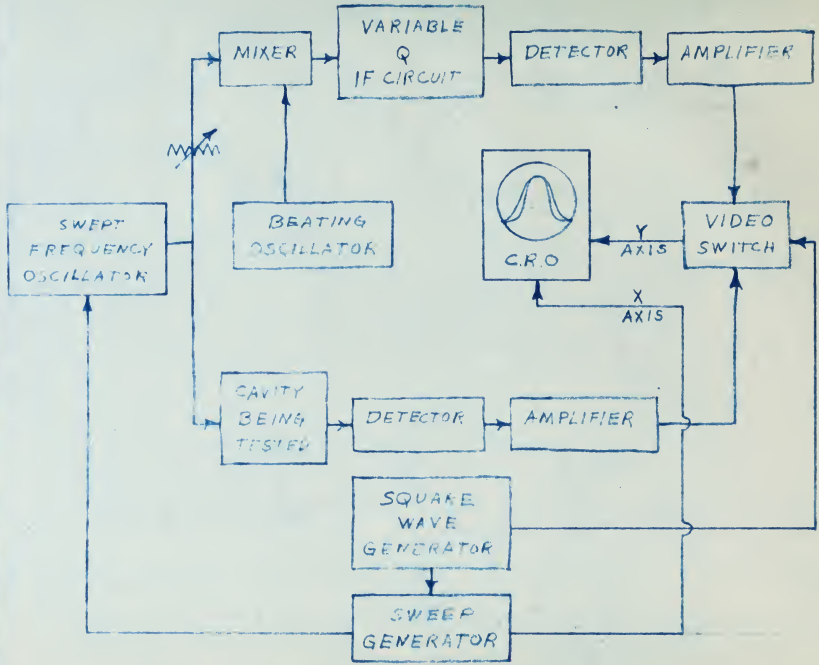
DETECTION LAW CHARACTERISTICS
OF A
1N32 CRYSTAL
R_L = 2.4 KΩ
R_L = 3.9 KΩ
R_L = 10 KΩ
f = 9400 MCPS
FOR R_L = 3.9 KΩ, VOLTAGE = 0.07 VOLTS AT -10 DBM
DETECTION LAW
3.0
2.0
1.0
0
-40
-30
-20
-10
0
POWER INPUT - DBM

FIGURE 2



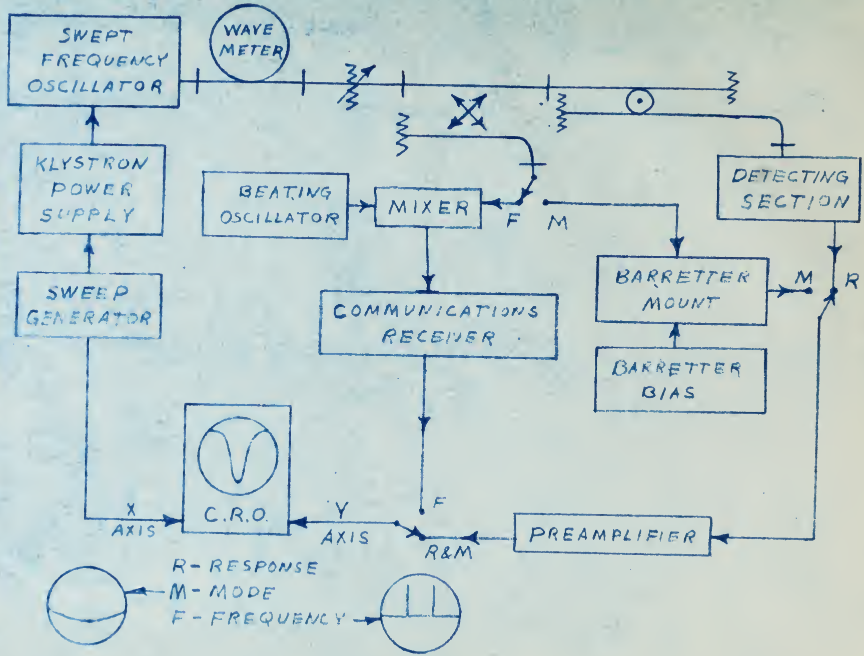
SINGLE DETECTOR COMPARISON SYSTEM

FIGURE 3



TWO DETECTOR COMPARISON SYSTEM

FIGURE 4



HETRODYNE MARKER SYSTEM

FIGURE 5

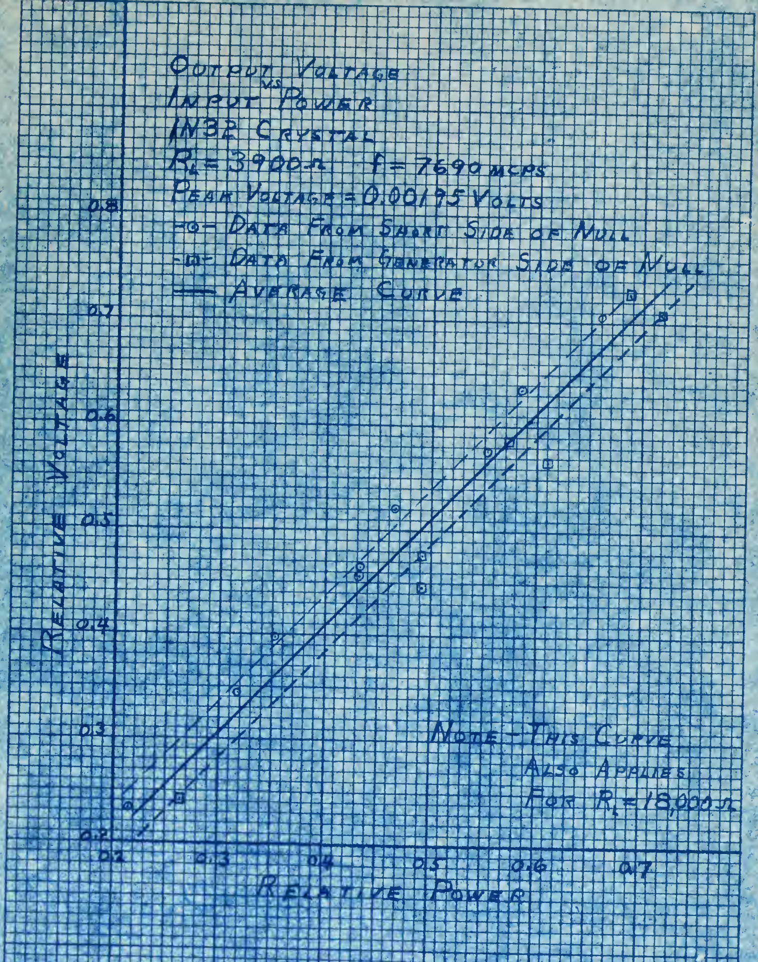

OUTPUT VOLTAGE
VS.
INPUT POWER
1N32 CRYSTAL
R_L = 3900 Ω f = 7690 MCPS
PEAK VOLTAGE = 0.00175 VOLTS
-⊙- DATA FROM SHORT SIDE OF NULL
-⊡- DATA FROM GENERATOR SIDE OF NULL
— AVERAGE CURVE
NOTE - THIS CURVE ALSO APPLIES FOR R_L = 18,000 Ω
RELATIVE VOLTAGE
RELATIVE POWER
0.2
0.3
0.4
0.5
0.6
0.7
0.8

FIGURE 6

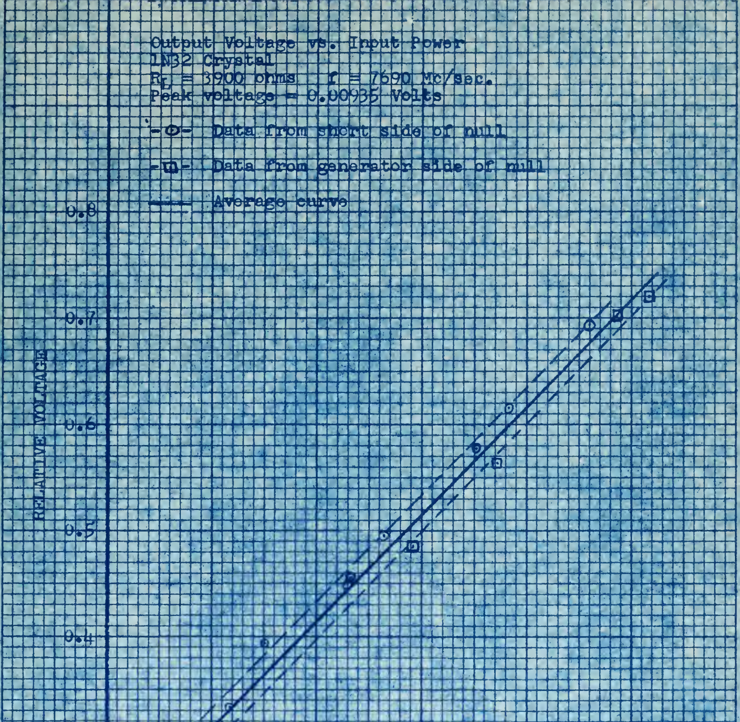

Output Voltage vs. Input Power
1N32 Crystal
R_L = 3900 ohms f = 7690 Mc/sec.
Peak voltage = 0.00935 Volts
Data from short side of null
Data from generator side of null
Average curve
RELATIVE VOLTAGE
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.2
0.3
0.4
0.5
0.6
0.7
RELATIVE POWER

FIGURE 7

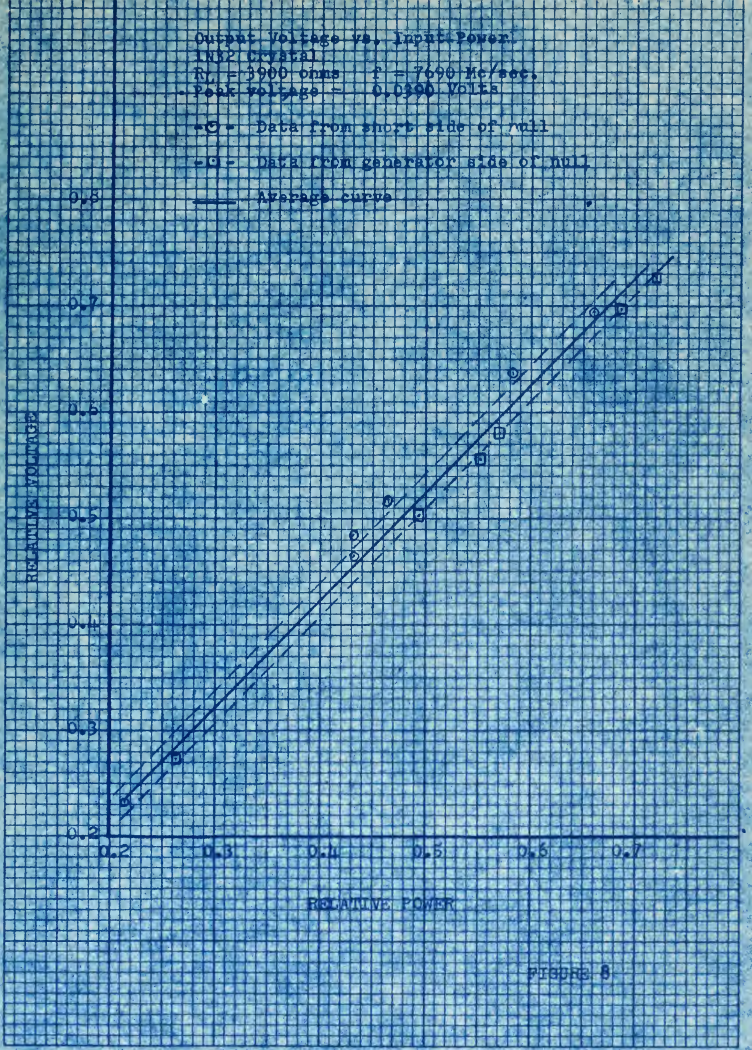

Output Voltage vs. Input Power.
1N32 Crystal
R_L = 3900 ohms   f = 7690 Mc/sec.
Peak voltage = 0.0390 Volts
-⊙- Data from short side of null
-⊡- Data from generator side of null
—— Average curve
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.2
0.3
0.4
0.5
0.6
0.7
RELATIVE VOLTAGE
RELATIVE POWER

FIGURE 8

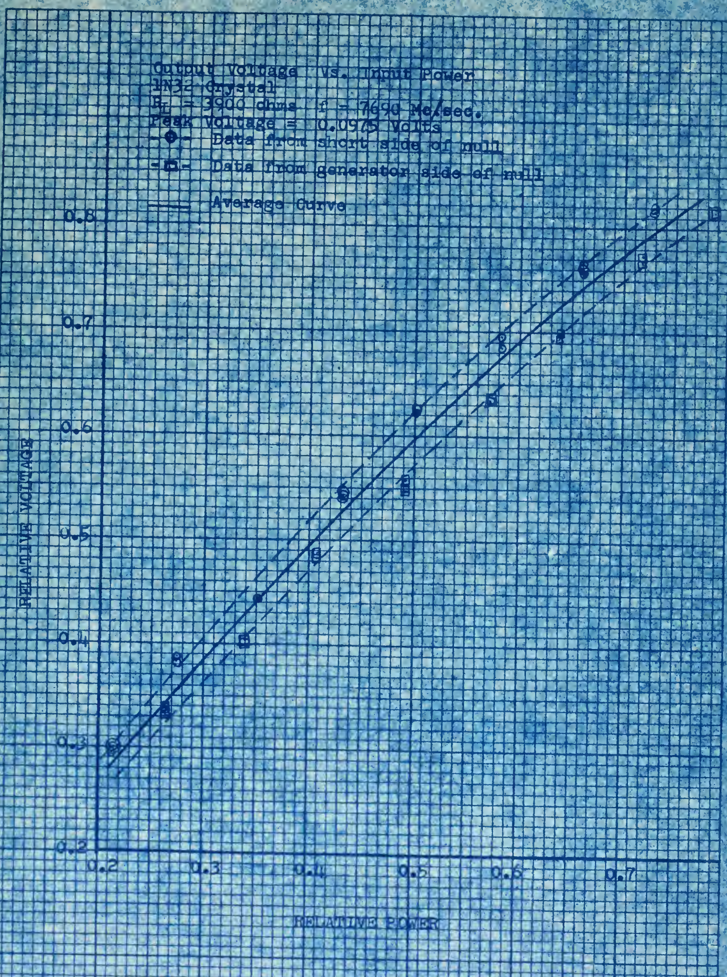
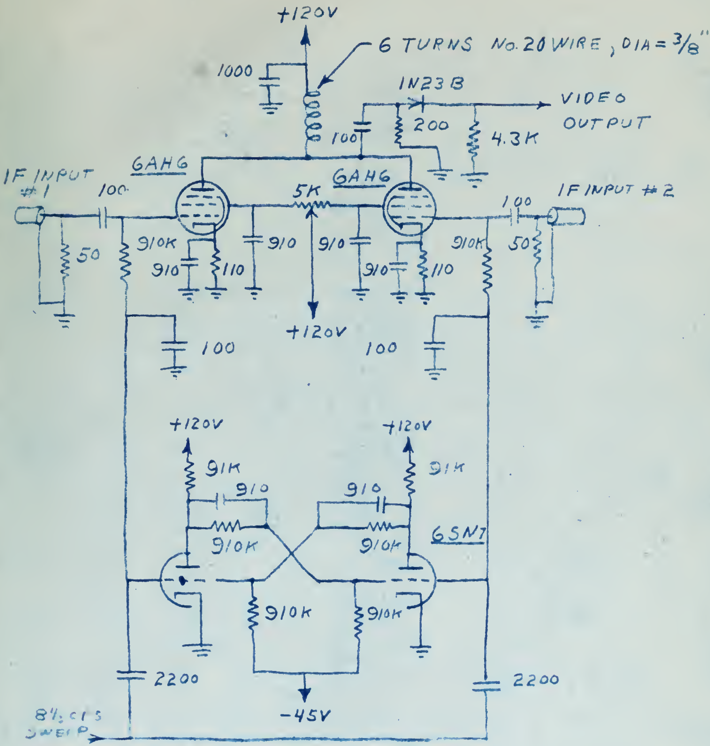
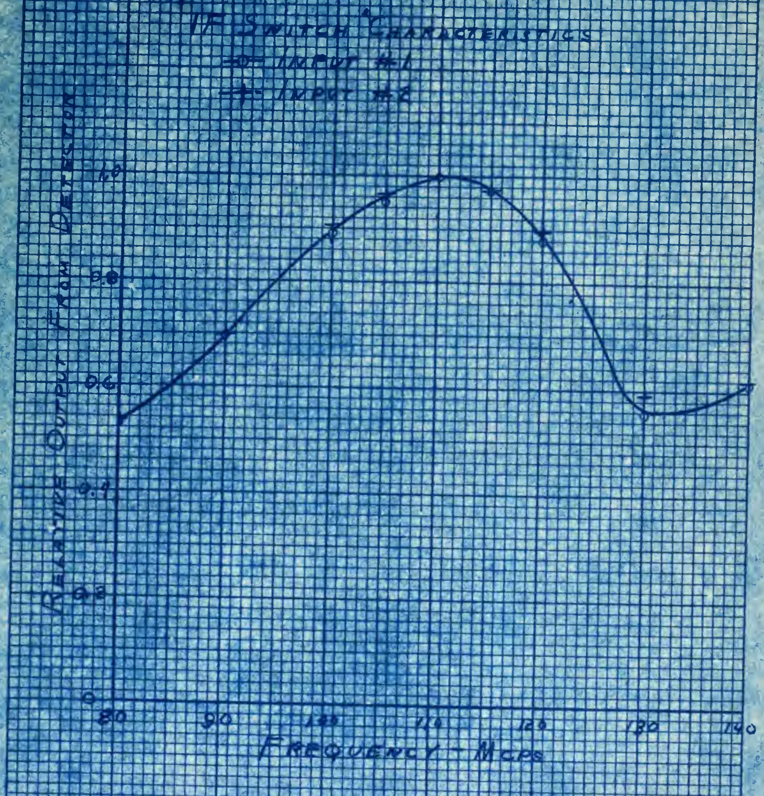

Output Voltage Vs. Input Power
1N32 Crystal
R_L = 3900 ohms f = 7690 Mc/sec.
Peak Voltage = 0.0975 Volts
Data from short side of null
Data from generator side of null
Average Curve
RELATIVE VOLTAGE
RELATIVE POWER
0.2
0.3
0.4
0.5
0.6
0.7
0.8

FIGURE 9



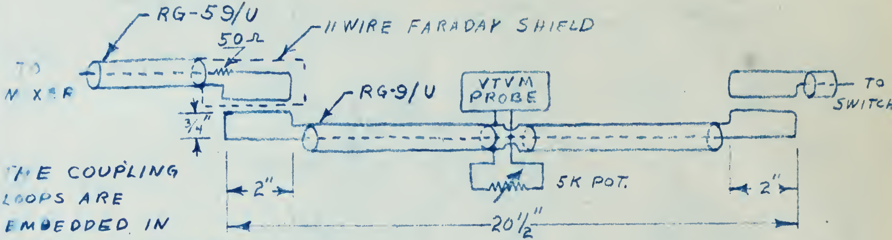
IF SWITCH

FIGURE - 10



IF SWITCH CHARACTERISTICS
—o— INPUT #1
—+— INPUT #2
RELATIVE OUTPUT FROM DETECTOR
1.0
0.8
0.6
0.4
0.2
0
80
90
100
110
120
130
140
FREQUENCY - MCPS

FIGURE - 11



THE COUPLING LOOPS ARE EMBEDDED IN POLYFOAM BLOCKS WITH 1" BETWEEN THE PLANES OF THE LOOPS

VARIABLE Q CIRCUIT $Q = 10-100$
$F = 105$ mcps

FIGURE - 12

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